# AUTOMATIC GENERALIZATION OF MAPS BY DIGITAL FILTERING

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**ABSTRACT**

Either paper or digital maps have limited capacity. Obviously, if too many objects are put on the map it becomes chaotic and unreadable. In order to avoid this high object density a process called generalization is needed. Untill now the generalization was made by the human interaction and intelligency. This paper introduces an automatic generalization technique based on digital elevation models and digital filtering methods. All map objects such as parcels, roads, rivers are on the Earth surface. If we change this surface by digital filters like low-pass filters, every object will change on it. Surface smoothing produces surface generalization, consequently every object on it will be generalized automatically.

***Index Terms—*** generalization, digital filter, digital elevation model

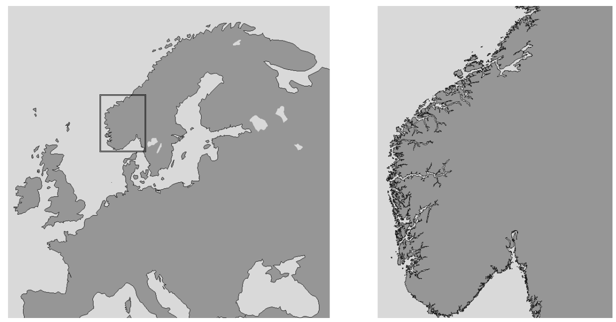

**Fig. 1**. On the left side figure the Scandinavian Penninsula can be seen on a small scaled map. The coastline is generalized in it, obviously. On the right side a south part of it was figured in a larger scaled map with less generalized and more accurate coastlines

## 1. INTRODUCTION

In traditional cartography there is a typicaly human made interaction the generalization. This process helps to avoid to produce overloaded, chaotic maps that contain too many objects. Large scale maps such as topographic maps contain every object on the Earth surface. In the GIS, the scale is very changable regarding the wide range functionalities of zoom in, zoom out, pan, and so on. If the scale becomes smaller (zoom out) the object density goes higher and the digital map can easily become overloaded, unreadable and chaotic.

It is well known since B. Mandelbrot that the length of coastlines depends on the scale. The extensions of map objects must depend on the scale. From the large scale to small one the generalization produces the proper object density. Look at the Fig.1 for the scale depedency of digital maps. Smaller scaled map requires generalized map content.

## 2. EARTH SURFACE AND DIGITAL ELEVATION MODELS

The real Earth surface is a complicated geometrical object that can not be described in analytical form. There are many objects on it such as parcels, buildings, rivers, roads, settlements and so on. If the surface is defined well, the position of every object on it can be properly described this surface.

Let us look at digital surface models and its mathematical background. This approach is based on the Sampling Theorem and the Fourier transform. A digital surface model is going to describe the analogue surface as accurate as possible. What does accurate mean in this context? The sampling rate determines the accuracy first of all. Sampling rate gives the regular grid size which is the distance between two points on the surface. A sampled surface consists of $x, y, z$ coordinates in every grid point.

Since the objects mentioned above are on the surface the accuracy of their position depends on the accuracy of the surface. This fact serves a correct mathematical model for the automatic generalization. Larger sampling distance in the resampling process gives more generalized map. Extremaly large sampling rate can remove complete objects that are negligible at a certain scale.

### 2.1. Outline of the sampling theorem

Let us name the interval between two sampling events *sampling period* and denote it $\tau$. The sampling period can be time

The elevation database (DDM-50) was provided by the Ministry of Defence Mapping Company

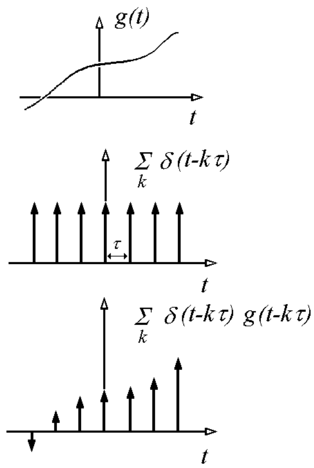


**Fig. 2**. The sketch of the ideal sampling process. Analogue function $g(t)$, Dirac-impulse train as a tool of sampling with $\tau$ sampling period, and the sampled function [2, 3]

dimension if we have time signals, but it can be distance dimension, if the sampling process is spatial such as digital photos, satellite images or digital elevation models.

Let $\tau$ be the sampling period. Let $g(t)$ denote a time function and the tool of the sampling process, which is a Dirac-impuls train, and finally the result of the sampling process, which is the digitised time function (Fig. 2). The sampling process can be defined as a product of the $g(t)$ function and the Dirac-impulse train.

$$g(t) \sum_{k=-\infty}^{\infty} \delta(t-k\tau) = \sum_{k=-\infty}^{\infty} g(k\tau)\delta(t-k\tau)$$

where the $Dirac-\delta$ is the following:

$$\delta(t) = 0, \qquad \text{if } t \neq 0 \qquad \text{and} \qquad \int_{-\infty}^{\infty} \delta(t)dt = 1$$

Remember the definition of the convolution of $f(t)$ and $g(t)$ functions:

$$h(t) = f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau.$$

Let au look at the following formula which is essential in understanding of the sampling process:

$$f(t) * g(t) = F(f) \cdot G(f)$$

$$F(f) * G(f) = f(t) \cdot g(t)$$

The convolution of two functions in the time domain is equal to the product of their Fourier transform in the frequency domain, and vice versa.

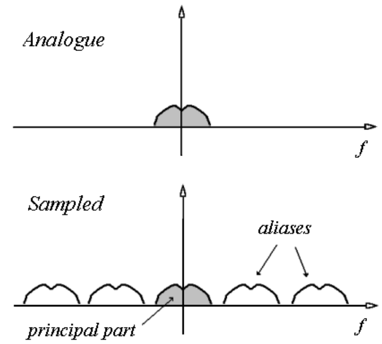


**Fig. 3**. The spectra of the sampled function becomes periodical [3]

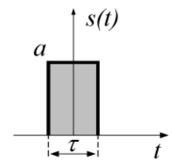


**Fig. 4**. The square function, $\tau$ width and $a$ height

Let us investigate the spectra of the analogue and the digitised functions. Let $G(f)$ denote the spectra of the analogue, and $G_d(f)$ the spectra of the digitised function. Regarding the properties of the Fourier transform of the Dirac-$\delta$ and the convolution, the spectra of the digitised (sampled) function is

$$G_d(f) = G(f) * \frac{1}{\tau} \sum_{k=-\infty}^{\infty} \delta(f - \frac{k}{\tau})$$

$$G_d(f) = \frac{1}{\tau} \sum_{k=-\infty}^{\infty} G(f - \frac{k}{\tau})$$

Compare the spectra of the analogue and digitised functions. There is a remarkable difference between them. The spectra of the sampled (digitised) function is not periodic, but the spectra of the sampled one becomes periodic because of sampling (Fig. 3).

Let us define the square function (Fig. 4), which is very important in this context.

$$\begin{aligned} s(t) &= a, \qquad \text{if } |t| < \tau/2 \\ &= 0, \qquad \text{else.} \end{aligned}$$

The Fourier transformed square function is the $sinc$ (sinus cardinalis) function (fig. 5).

$$S(f) = \int_{-\infty}^{\infty} s(t)e^{-2\pi ift}dt = a\int_{-\tau/2}^{\tau/2} e^{-2\pi ift}dt = a\frac{\sin \pi f\tau}{\pi f\tau}$$

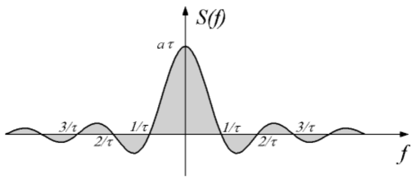


**Fig. 5**. The Fourier transformed square function is the so called $sinc$ function. If there is a square function in the frequency domain, the Fourier transformed square function is a $sinc$ function in the time domain and vice versa

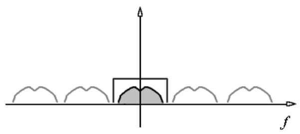


**Fig. 6**. In order to remove aliases and to preserve the principal part of the spectra, truncate it with a square function [3, 2]

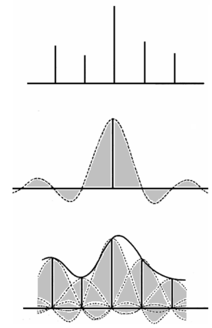

**Fig. 7**. The process of the signal recovery. The recovered curve is made by the product of the samples and the $sinc$ functions at proper arguments [2]

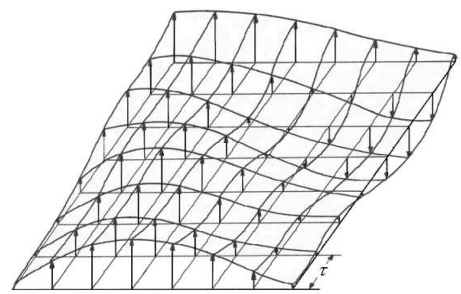


**Fig. 8**. A surface sampled by a 2D Dirac-$\delta$ series [2]

Let $s(t)$ and $g(t)$ denote functions in the time domain and $S(f)$ and $G(f)$ functions their spectras in the frequency domain.

$$S(f) \cdot G(f) = s(t) * g(t)$$

Consequently, the product of a spectra $G(f)$ with a square-function $S(f)$ is equal to the convolution of $sinc(t)$ and $g(t)$.

### 2.2. Recovery of the analogue function

If you take sampling theory into account while you are digitising, the sampled data series is equivalent to the analogue one. In this case there is no data waste. How is it possible?

Regarding the properties of the convolution and the inverse Fourier transformed square function, the product of the spectra and square function with $\tau$ height and $1/\tau$ width in the frequency domain is equivalent to the convolution of the original function and the $sinc$ function in the time domain.

Let $s(t)$ be the square function. In order to keep the principal part of the spectra only, cut the outside parts, the so called aliases (Fig. 6):

$$G_d(f)\tau \cdot s(f\tau) = G(f)$$

In this way, the spectra of the analogue and digitised dataset becomes the same. Consequently, the analogue dataset can be recovered from the digitised dataset without any waste. Remember, the sampling process is made in time domain, so we must know what happens in the time domain if we truncate the aliases in the frequency domain. Let us have the invert Fourier transformed truncated spectra

$$\mathcal{F}^{-1}\{G_d(f)\} = \sum_{k=-\infty}^{\infty} g(k\tau)\delta(t - k\tau)$$

taking into account the followings:

$$\mathcal{F}^{-1}\{\tau \cdot s(f\tau)\} = sinc(t/\tau - k).$$

The recovery of the original signal can be made by the next step:

$$\left(\sum_{k=-\infty}^{\infty} g(t)\delta(t - k\tau)\right) sinc(t/\tau) = \sum_{k=-\infty}^{\infty} g(k\tau) sinc(t/\tau - k)$$

The recovered values in the sampling places are constructed by the multiplication of samples and values of the $sinc$ function with proper argument (Fig. 7). So the recovered values are exactly the same as the values of the analogue function in the same argument. The recovered values can be computed by the multiplication of samples and values of $sinc$ function with proper argument in arbitrary arguments, in any place.

### 2.3. Sampling in 2D

If we apply the sampling process to a surface, the sampling tool will be like a yogi's nailed bed (Fig. 8). Let us have a $\tau$ sampling distance for defining a grid. Every grid point has $x, y, z$ coordinates. This is the digital elevation model (DEM). The resolution of the DEM depends on $\tau$. As mentioned above every object on the Earth properly defined by a grid. There are other ways of representing the terrain elevation [5, 1, 4], but the current one is precisely adjustable for different scales, consequently for the automatic generalization.

## 3. EXAMPLES

Let us look at some examples where the downsampling technique is applied for generalizing an elevation database (Fig. 9, 10, 11, 12). The original data source was a $50 \times 50$m elevation grid. From this database a pretty 3D view was generated (Fig. 9). A new sampling period was set up 500m and the downsampled 3D view can be seen on Fig. 10. Shaded elevation was generated both from the original DEM (Fig. 11) and downsampled one as well (Fig. 12).

## 4. CONCLUSION

Some image processing technique such as low-pass filtering is a very good method for automatic generalisation of the Earth surface, consequently each map object on it. Regarding the theoretical background of the sampling theory the downsampling produces smoothed dataset. Instead of using simple low-pass filters on the original database downsampling produces smoothed digital elevation model also, but smaller sized database.

## 5. REFERENCES


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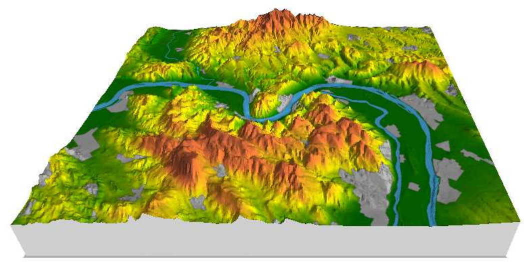

**Fig. 9**. A 3D elevation model with 50m grid size before resampling

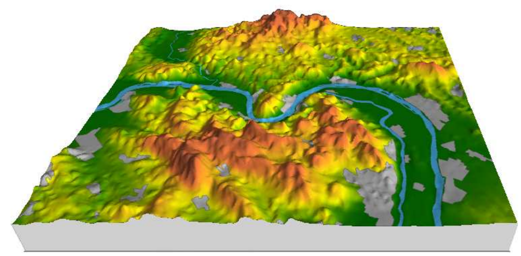

**Fig. 10**. The 3D elevation model with 500m grid size after downsampling

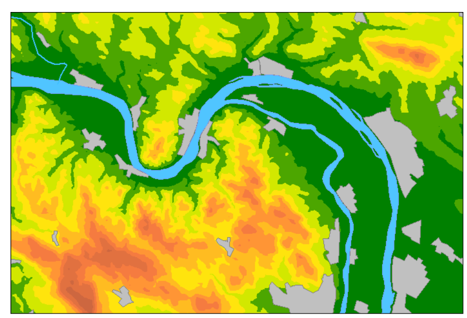

**Fig. 11**. Coloured contours with 50m grid size before resampling

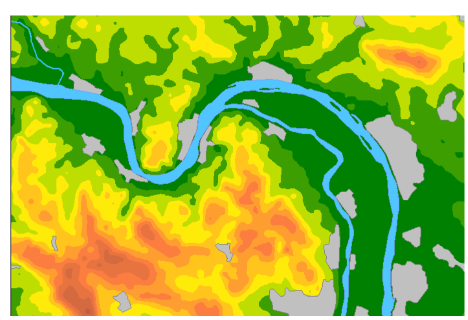

**Fig. 12**. Coloured contours with 500m grid size after downsampling